Bayesian state-space models for affective dynamics

Tom Lodewyckx, Francis Tuerlinckx, Peter Kuppens
Quantitative psychology & individual differences
KULeuven

Nicholas Allen
University of Melbourne

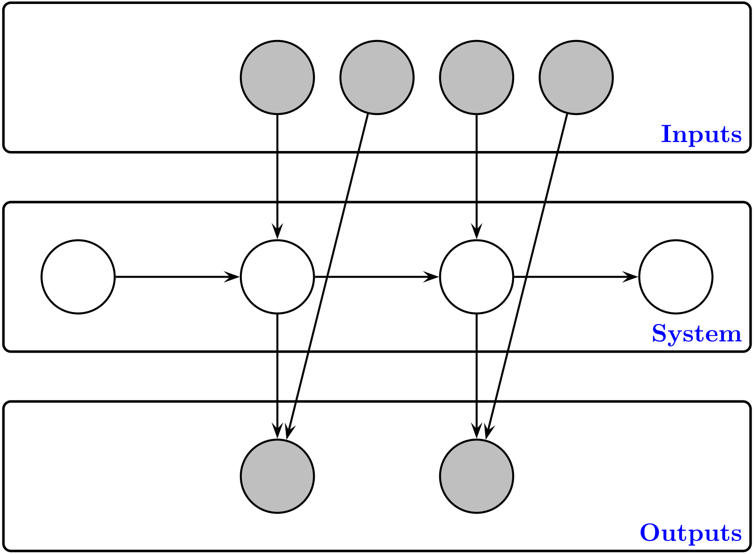
Lisa Sheeber
Oregon Research Institute

OUTLINE

OUTLINE

Linear dynamical system theory

- *Inputs (observed):* control / noise / demographic variables
- *System (unobserved):* latent states with dynamical dependencies
- *Outputs (observed):* measurements of interest


Inputs
System
Outputs

OUTLINE

Application fields

- Engineering
- Econometrics
- Ecological research

Application fields

- Engineering
- Econometrics
- Ecological research

Model structure

- Transition equation: latent dynamical system
- Observation equation: observed variables

Transition equation

$$Y_t \sim \text{Gaussian}\left(\Psi\Theta_t + \Gamma X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$$

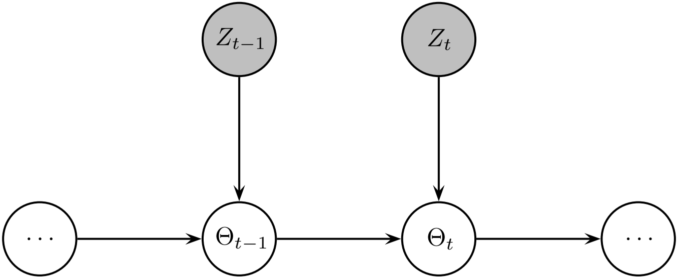
- Θ_t = states
- Θ_{t-1} = states at previous observation moment
- Z_t = state covariates

Transition equation

$$Y_t \sim \text{Gaussian}\left(\Psi\Theta_t + \Gamma X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$$

- Θ_t = states
- Θ_{t-1} = states at previous observation moment
- Z_t = state covariates
- Φ = transition matrix (autocorrelations & cross-lagged relations)
- Δ = state covariate regression coefficient matrix
- Σ_η = innovation variance/covariance matrix


Z_{t-1}
Z_t
$\cdots$
Θ_{t-1}
Θ_t
$\cdots$

Observation equation

$$Y_t \sim \text{Gaussian}\left(\Psi\Theta_t + \Gamma X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$$

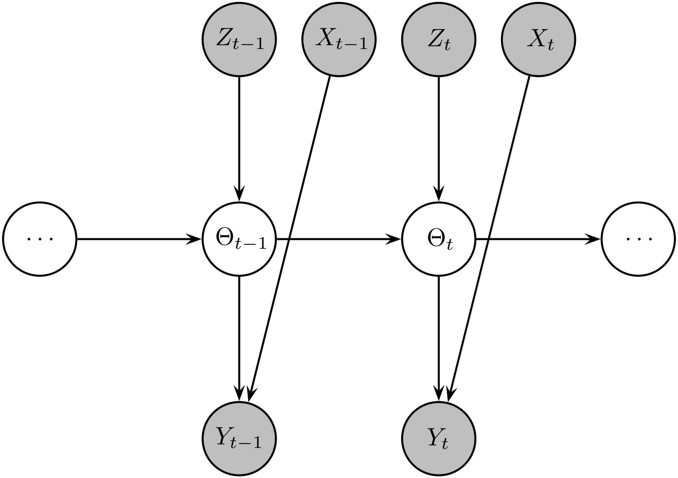
- Y_t = observations
- Θ_t = states
- X_t = observation covariates

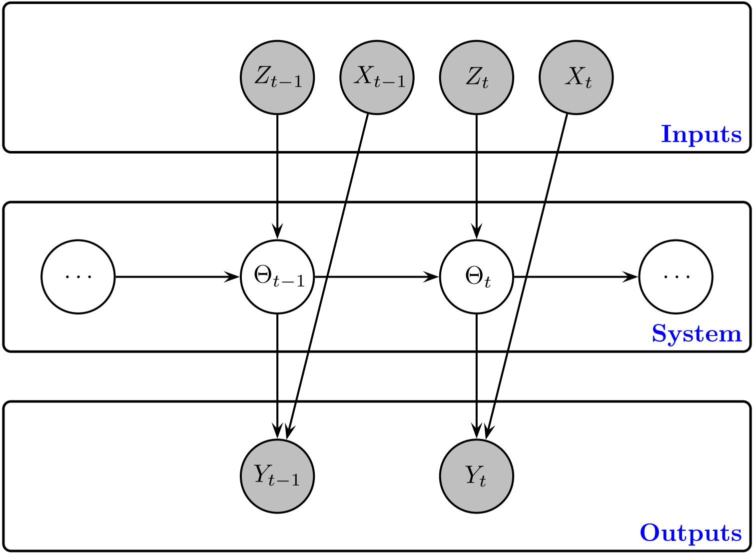
Observation equation

$$Y_t \sim \text{Gaussian}\left(\Psi\Theta_t + \Gamma X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$$

- Y_t = observations
- Θ_t = states
- X_t = observation covariates
- Ψ = design matrix (mapping $\Theta_t \rightarrow Y_t$)
- Γ = observation covariate regression coefficient matrix
- Σ_ϵ = observation error variance/covariance matrix


Z_{t-1}
X_{t-1}
Z_t
X_t
$\cdots$
Θ_{t-1}
Θ_t
$\cdots$
Y_{t-1}
Y_t


Z_{t-1}
X_{t-1}
Z_t
X_t
Inputs
$\cdots$
Θ_{t-1}
Θ_t
$\cdots$
System
Y_{t-1}
Y_t
Outputs

OUTLINE

1. **State-space models**
   - Linear dynamical systems
   - Multivariate linear Gaussian state-space model
   - *Regime-switching MLGSS model*
2. Bayesian estimation
3. Application
4. Discussion

Regime-switching MLGSS model

$$Y_t \sim \text{Gaussian}\left(\boldsymbol{\Psi}_{\mathbf{t}}\Theta_t + \boldsymbol{\Gamma}_{\mathbf{t}}X_t, \Sigma_{\epsilon t}\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\boldsymbol{\Phi}_{\mathbf{t}}\Theta_{t-1} + \boldsymbol{\Delta}_{\mathbf{t}}Z_t, \Sigma_{\eta t}\right)$$

Regime-switching MLGSS model

$$Y_t \sim \text{Gaussian}\left(\mathbf{\Psi_t}\Theta_t + \mathbf{\Gamma_t}X_t, \Sigma_{\epsilon t}\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\mathbf{\Phi_t}\Theta_{t-1} + \mathbf{\Delta_t}Z_t, \Sigma_{\eta t}\right)$$

$$\left(\begin{array}{c} \Psi_t, \Gamma_t \\ \Phi_t, \Delta_t \end{array}\right) = \begin{cases} \left(\begin{array}{c} \Psi_1, \Gamma_1 \\ \Phi_1, \Delta_1 \end{array}\right) & \text{if } R_t = 1 \\ \qquad \cdots \\ \left(\begin{array}{c} \Psi_r, \Gamma_r \\ \Phi_r, \Delta_r \end{array}\right) & \text{if } R_t = r \end{cases}$$

OUTLINE

Gibbs sampling

- Bayesian estimation method, iterative Monte Carlo simulation
- Each parameter has conditional sampling distribution
- Blocking of parameters efficient for certain models
  (no WinBUGS ☹)

Gibbs sampling

- Bayesian estimation method, iterative Monte Carlo simulation
- Each parameter has conditional sampling distribution
- Blocking of parameters efficient for certain models (no WinBUGS ☹)

Blocked Gibbs sampler

1. Sampling distribution Θ
2. Sampling distribution $\Psi_1, \ldots, \Psi_r, \Gamma_1, \ldots, \Gamma_r, \Sigma_\epsilon$
3. Sampling distribution $\Phi_1, \ldots, \Phi_r, \Delta_1, \ldots, \Delta_r, \Sigma_\eta$

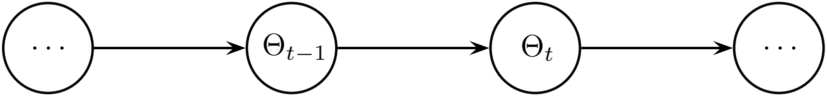
Blocked Gibbs sampler

1. Sampling distribution Θ
   $\longrightarrow$ Forward-filtering backward sampling
   (Carter & Kohn, 1994, 1996)
2. Sampling distribution $\Psi_1, \ldots, \Psi_r, \Gamma_1, \ldots, \Gamma_r, \Sigma_\epsilon$
3. Sampling distribution $\Phi_1, \ldots, \Phi_r, \Delta_1, \ldots, \Delta_r, \Sigma_\eta$

$\Rightarrow$ **Kalman filter** $\Rightarrow$

$$\cdots \longrightarrow \Theta_{t-1} \longrightarrow \Theta_t \longrightarrow \cdots$$

$\Rightarrow$ **Kalman filter** $\Rightarrow$



$\Leftarrow$ **Backward-sampling** $\Leftarrow$

Blocked Gibbs sampler

1. Sampling distribution Θ
2. Sampling distribution $\Psi_1, \dots, \Psi_r, \Gamma_1, \dots, \Gamma_r, \Sigma_\epsilon$
   $\longrightarrow$ Method of decomposition (Zellner, 1962)
3. Sampling distribution $\Phi_1, \dots, \Phi_r, \Delta_1, \dots, \Delta_r, \Sigma_\eta$
   $\longrightarrow$ Method of decomposition (Zellner, 1962)

Parameters observation equation

$\Sigma_\epsilon \longrightarrow$ Inverse Wishart

Parameters observation equation

$\Sigma_\epsilon \longrightarrow$ Inverse Wishart

$\Psi_1, \ldots, \Psi_r \mid \Sigma_\epsilon \longrightarrow$ Multivariate Gaussian

Parameters observation equation

$\Sigma_\epsilon \rightarrow$ Inverse Wishart

$\Psi_1, \ldots, \Psi_r \mid \Sigma_\epsilon \rightarrow$ Multivariate Gaussian

$\Delta_1, \ldots, \Delta_r \mid \Sigma_\epsilon, \Psi_1, \ldots, \Psi_r \rightarrow$ Multivariate Gaussian

Parameters observation equation

$\Sigma_\epsilon \longrightarrow$ Inverse Wishart

$\Psi_1, \ldots, \Psi_r \mid \Sigma_\epsilon \longrightarrow$ Multivariate Gaussian

$\Delta_1, \ldots, \Delta_r \mid \Sigma_\epsilon, \Psi_1, \ldots, \Psi_r \longrightarrow$ Multivariate Gaussian

Parameters transition equation

$\Sigma_\eta \longrightarrow$ Inverse Wishart

$\Phi_1, \ldots, \Phi_r \mid \Sigma_\eta \longrightarrow$ Multivariate Gaussian

$\Gamma_1, \ldots, \Gamma_r \mid \Sigma_\eta, \Phi_1, \ldots, \Phi_r \longrightarrow$ Multivariate Gaussian

OUTLINE

Emotion regulation

- High: emotions are adapted to the social context

  Low: emotions are insensitive to the context

Emotion regulation

- High: emotions are adapted to the social context

  Low: emotions are insensitive to the context
- Baseline respiratory sinus arrhythmia (QRSA)

  Butler, Wilhelm & Gross (2006); Gyurak & Ayduk (2008)

Emotion regulation

- High: emotions are adapted to the social context

  Low: emotions are insensitive to the context
- Baseline respiratory sinus arrhythmia (QRSA)

  Butler, Wilhelm & Gross (2006); Gyurak & Ayduk (2008)

Emotional inertia

- High: emotions don't change much over time

  Low: emotions constantly fluctuate over time

Emotion regulation

- High: emotions are adapted to the social context

  Low: emotions are insensitive to the context
- Baseline respiratory sinus arrhythmia (QRSA)

  Butler, Wilhelm & Gross (2006); Gyurak & Ayduk (2008)

Emotional inertia

- High: emotions don't change much over time

  Low: emotions constantly fluctuate over time
- Autocorrelation of emotional process (ϕ_{em})

  Kuppens, Allen & Sheeber (submitted)

Oregon adolescent interaction data

- 69 normal & 72 depressed adolescents
- Adolescent, father and mother in laboratory
- Problem solving task (e.g. pocket money)
- One measure/second during 18 minutes ($n = 1080$)

Oregon adolescent interaction data

- 69 normal & 72 depressed adolescents
- Adolescent, father and mother in laboratory
- Problem solving task (e.g. pocket money)
- One measure/second during 18 minutes ($n = 1080$)

Measurements

- QRSA (emotion regulation)

Oregon adolescent interaction data

- 69 normal & 72 depressed adolescents
- Adolescent, father and mother in laboratory
- Problem solving task (e.g. pocket money)
- One measure/second during 18 minutes ($n = 1080$)

Measurements

- QRSA (emotion regulation)
- HR_t (heart rate, "Fight-or-flight")
- RSA_t (respiratory sinus arrhythmia, "Rest-and-digest")

Oregon adolescent interaction data

- 69 normal & 72 depressed adolescents
- Adolescent, father and mother in laboratory
- Problem solving task (e.g. pocket money)
- One measure/second during 18 minutes ($n = 1080$)

Measurements

- QRSA (emotion regulation)
- HR_t (heart rate, "Fight-or-flight")
- RSA_t (respiratory sinus arrhythmia, "Rest-and-digest")
- AdAnger_t (anger of adolescent)

Basic MLGSS Model

- $[Y_{\text{HR}} \;\; Y_{\text{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t, \Sigma_\epsilon\right)$

Basic MLGSS Model

- $[Y_{\mathrm{HR}} \;\; Y_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t, \Sigma_\epsilon\right)$
  $[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(\boldsymbol{\Phi}[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_{t-1}, \Sigma_\eta\right)$

Basic MLGSS Model

- $[Y_{\text{HR}} \;\; Y_{\text{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t, \Sigma_\epsilon\right)$
  $[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t \sim \text{Gaussian}\left(\boldsymbol{\Phi}[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_{t-1}, \Sigma_\eta\right)$

Results

	Normal	Depressed
$\hat{\phi}_{\text{HR}}$	.79 (.08)	.81 (.08)

Basic MLGSS Model

- $[Y_{\mathrm{HR}} \;\; Y_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t, \Sigma_\epsilon\right)$
  $[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(\boldsymbol{\Phi}[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_{t-1}, \Sigma_\eta\right)$

Results

	Normal	Depressed
$\hat{\phi}_{\mathrm{HR}}$	.79 (.08)	.81 (.08)
r($\hat{\phi}_{\mathrm{HR}}$,QRSA)	-.28	-.64

Regime-switching MLGSS Model

- $[Y_{\text{HR}} \;\; Y_{\text{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t, \Sigma_\epsilon\right)$

  $[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t \sim \text{Gaussian}\left(\Phi_t[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_{t-1}, \Sigma_\eta\right)$

  $$\Phi_t = \begin{cases} \mathbf{\Phi}_{\textbf{[no anger]}} \text{ if AdAnger}_t = 0 \\ \mathbf{\Phi}_{\textbf{[anger]}} \text{ if AdAnger}_t = 1 \end{cases}$$

Regime-switching MLGSS Model

- $[Y_{\text{HR}} \;\; Y_{\text{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t, \Sigma_\epsilon\right)$

  $[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_t \sim \text{Gaussian}\left(\Phi_t[\Theta_{\text{HR}} \;\; \Theta_{\text{RSA}}]_{t-1}, \Sigma_\eta\right)$

  $$\Phi_t = \begin{cases} \boldsymbol{\Phi}_{\textbf{[no anger]}} & \text{if AdAnger}_t = 0 \\ \boldsymbol{\Phi}_{\textbf{[anger]}} & \text{if AdAnger}_t = 1 \end{cases}$$

Results

	Normal	Depressed
$\hat{\phi}_{\text{HR[no anger]}}$	.79 (.08)	.80 (.08)
$\hat{\phi}_{\text{HR[anger]}}$	.73 (.16)	.78 (.11)

Regime-switching MLGSS Model

- $[Y_{\mathrm{HR}} \;\; Y_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(I_2[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t, \Sigma_\epsilon\right)$
  $[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_t \sim \text{Gaussian}\left(\Phi_t[\Theta_{\mathrm{HR}} \;\; \Theta_{\mathrm{RSA}}]_{t-1}, \Sigma_\eta\right)$
  $\Phi_t = \begin{cases} \mathbf{\Phi}_{\text{[no anger]}} & \text{if AdAnger}_t = 0 \\ \mathbf{\Phi}_{\text{[anger]}} & \text{if AdAnger}_t = 1 \end{cases}$

Results

	Normal	Depressed
$\hat{\phi}_{\text{HR[no anger]}}$	.79 (.08)	.80 (.08)
$\hat{\phi}_{\text{HR[anger]}}$	.73 (.16)	.78 (.11)
r($\hat{\phi}_{\text{HR[no anger]}}$,QRSA)	-.26	-.67
r($\hat{\phi}_{\text{HR[anger]}}$,QRSA)	-.10	-.65

Summary

- No difference in heart rate inertia between normal and depressed subjects

Summary

- No difference in heart rate inertia between normal and depressed subjects
- Negative relation between heart rate inertia and QRSA: The higher (lower) the degree of emotion regulation, the weaker (stronger) the emotional inertia

Summary

- No difference in heart rate inertia between normal and depressed subjects
- Negative relation between heart rate inertia and QRSA: The higher (lower) the degree of emotion regulation, the weaker (stronger) the emotional inertia (asymmetry normal & depressed?)

OUTLINE

Modeling issues

- Hierarchical modeling (individual & group differences)
- Prior distributions
- Latent regime variable
- Hypothesis testing

Modeling issues

- Hierarchical modeling (individual & group differences)
- Prior distributions
- Latent regime variable
- Hypothesis testing

Conceptual issues

- Factor structures in design matrix
- Non-Gaussian errors
- Lagging factor

Conclusion

1. Framework useful for latent psychophysiological dynamics

Conclusion

1. Framework useful for latent psychophysiological dynamics
2. Bayesian estimation using MCMC methods

Conclusion

1. Framework useful for latent psychophysiological dynamics
2. Bayesian estimation using MCMC methods
3. Hierarchical model more elegant

Conclusion

1. Framework useful for latent psychophysiological dynamics
2. Bayesian estimation using MCMC methods
3. Hierarchical model more elegant
4. Various other model extensions to be considered

Conclusion

1. Framework useful for latent psychophysiological dynamics
2. Bayesian estimation using MCMC methods
3. Hierarchical model more elegant
4. Various other model extensions to be considered

Questions?